Unit 2: Kinematics

Problem

A particle starts from origin at t = 0 s with a velocity of 10 $\hat{j}$ m s^{-1} and moves in the X-Y plane with a constant acceleration of $8\hat{\imath} + 2\hat{j}\ m\ s^{-2}$.

a. At what time is the x-coordinate of the particle 16 m? What is the y-coordinate of the particle at that time?

b. What is the speed of the particle at that time?

Solution:

Initial velocity of the particle, u = 10 $\hat{j}$ m s^{-1}

Particle moving in X-Y plane with constant acceleration, a = $8\hat{\imath} + 2\hat{j}\ m\ s^{-2}$

If $\vec{r}$ is the position vector of the particle at time t, then,

$$\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$=10\ \hat{j} + \frac{1}{2}(8\hat{\imath} + 2\hat{j})t^2$$

$$\vec{r} = 4\,t^2\hat{\imath} + (10t + t^2)\hat{\jmath} \qquad (1)$$

If (x,y) are the coordinates of the particle at time t, then,

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} \qquad (2)$$

From equation (1) and (2), we have

$$x\hat{\imath} + y\hat{\jmath} = 4\,t^2\hat{\imath} + (10t + t^2)\hat{\jmath}$$

Now comparing the coefficient of $\hat{\imath}$ and $\hat{\jmath}$, we have,

$x = 4t^2$

$y = 10t + t^2$

when x= 16 m, the above equation becomes

$16 = 4t^2$

$t^2 = 4$

$t = 2s$

when t = 2s, then $y = 10 \times 2 + 2^2$

$= 20 + 4$

$= 24$ m

b. If $\vec{v}$ is the velocity of particle at time t, the,

$$\vec{v} = \vec{u} + \vec{a}t$$

$$= 10\,\hat{j} + (8\hat{\imath} + 2\hat{j})t$$

$$= 8\,t\,\hat{\imath} + (10 + 2t)\hat{j}$$

At t = 2s,

$$\vec{v} = 8 \times 2\,\hat{\imath} + (10 + 2 \times 2)\hat{j}$$

$$= 16\,\hat{\imath} + 14\,\hat{j}$$

$$v = \sqrt{16^2 + 14^2}$$

$$= \sqrt{256 + 196}$$

$$= \sqrt{452}$$

$$= 21.26 \text{ m } s^{-1}$$

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